Exam. Code : 103202
Subject Code : 1028

B.A./B.Sc. Semester—II
MATHEMATICS
(Calculus)
Paper—II

Time Allowed—3 Hours] [Maximum Marks—50

Note :— Attempt **FIVE** questions in all selecting at least **TWO** questions from each section. All questions carry equal marks.

SECTION—A

I. (a) Show that $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^3 + y^3}{x - y}$ does not exist.

(b) Show that the function f, where

$$f(x, y) = \begin{cases} xy\dfrac{x^2 - y^2}{x^2 + y^2}, & \text{if } x^2 + y^2 \neq 0 \\ 0, & \text{if } x = y = 0 \end{cases}$$

is differentiable at origin. 5,5

II. (a) State and prove Young's Theorem.

(b) If $z = x^3 - xy + y^3$, $x = r\cos\theta$, $y = r\sin\theta$, find $\dfrac{\partial z}{\partial r}, \dfrac{\partial z}{\partial \theta}$. 5,5

III. (a) Show that the function $f(x, y) = 2x^4 - 3x^2y + y^2$ has neither a maximum nor a minimum value at (0, 0), where $f_{xx}f_{yy} - (f_{xy})^2 = 0$.

(b) Expand $x^4 + x^2y^2 - y^4$ about the point (1, 1) up to terms of the second degree. 5,5

IV. (a) The roots of the equation

$$(\lambda - x)^3 + (\lambda - y)^3 + (\lambda - z)^3 = 0$$

in λ are u, v, w. Prove that :

$$\frac{\partial(u, v, w)}{\partial(x, y, z)} = -2\frac{(y-z)(z-x)(x-y)}{(v-w)(w-u)(u-v)}.$$

(b) Find the envelope of the family of lines

$$x\cos^3\theta + y\sin^3\theta = a,$$

where θ is parameter. 5,5

V. (a) Find the envelope of the ellipses having the axes of co-ordinates as principal axes and sum of their semi-axis is constant.

(b) Let z be a function of x and y. Prove that if $x = e^u + e^{-v}$, $y = e^{-u} - e^v$ then

$$\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y}.$$ 5,5

SECTION—B

VI. (a) Show that $\int_0^1 dx \int_0^1 \frac{x^2 - y^2}{x^2 + y^2} dy = \int_0^1 dy \int_0^1 \frac{x^2 - y^2}{x^2 + y^2} dx$.

(b) Evaluate $\iiint_E z^2 \, dx\, dy\, dz$ taken over the region common to the surfaces $x^2 + y^2 + z^2 = a^2$, and $x^2 + y^2 = ax$. 5,5

VII. (a) Compute $I = \iiint \sqrt{1 - \frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2}}\, dx\, dy\, dz$ taken over the region $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.

(b) Evaluate $\int_0^{\pi}\int_0^{\pi} |\cos(x + y)|\, dx\, dy$. 5,5

VIII. (a) Change the order of integration in

$$\int_0^1 \int_0^{\sqrt{2 - x^2}} \frac{x}{\sqrt{x^2 + y^2}}\, dx\, dy$$ and hence evaluate.

(b) Show that :

$$\iiint_E (ax + by + cz)^2\, dx\, dy\, dz = \frac{4}{15}\pi\,(a^2 + b^2 + c^2)$$

where domain E is the sphere $x^2 + y^2 + z^2 \le 1$.

5,5

IX. (a) Compute the surface area S of the sphere $x^2 + y^2 + z^2 = a^2$.

(b) Evaluate $\iiint_R dx\, dy\, dz$ where R is the region common to the cylinders $x^2 + y^2 = a^2$ and $x^2 + z^2 = a^2$.

5,5

X. (a) Compute the area bounded by the parabolas $y^2 = ax$, $y^2 = bx$, $x^2 = py$, $x^2 = qy$, where $0 < a < b$, and $0 < p < q$.

(b) Evaluate $\iint_E \sqrt{a^2 - x^2 - y^2}\, dx\, dy$, where E is the region bounded by the circle $x^2 + y^2 = ax$. 5,5